CONSTRUCTION OF $7 \sqcup 1\langle 1\langle 11\rangle\rangle$ IN DEGREE 8

ERWAN BRUGALLÉ

Abstract. We construct an $M-2$ real algebraic curve of degree 8 in $\mathbb{R}P^2$ realizing the real scheme $7 \sqcup 1\langle 1\langle 11\rangle\rangle$, answering to a question by Stepan Orevkov. Even if it is not visible in the proof presented here, we were guided in our construction by a tropical interpretation of Orevkov's inital construction [Ore02].

The existence of a real algebraic curve of degree 8 realizing the real scheme $7\sqcup 1\langle 1\langle 11\rangle\rangle$ is contained in the following proposition.

Proposition 1. *For any $0 \leq \alpha \leq 7$, $0 \leq \beta \leq 2$ and $0 \leq \gamma \leq 11$, there exists a real algebraic curve of degree 8 realizing the real schemes $\alpha \sqcup 1\langle \beta \sqcup 1\langle \gamma\rangle\rangle$ and $\gamma \sqcup 1\langle \beta \sqcup 1\langle \alpha\rangle\rangle$.*

Proof. We first construct by patchworking a maximal real algebraic curve of bidegree $(4,0)$ in Σ_4 as depicted in Figure 1. According to [Che02] (see also [Shu99]), this implies the existence of an M-curve of degree 8 realizing the real scheme $7 \sqcup 1\langle 2 \sqcup 1\langle 11\rangle\rangle$ (see [Ore02] for the original construction) The two trigonal curves are constructed in Lemma 2 and 3. The proposition follows from the construction of the pieces used in the patchworking (see [Ore03] for the justification that one can contract any ovals of a trigonal curve). □

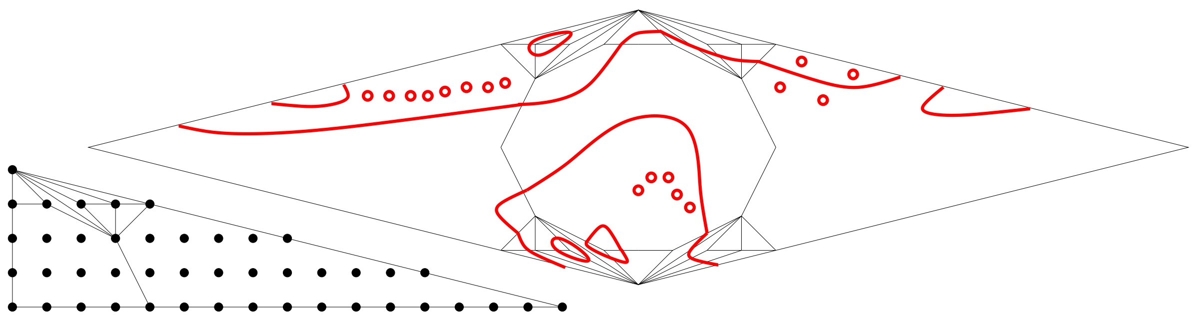


Figure 1

Lemma 2. *There exists a real algebraic curve whose Newton Polygon and chart are depicted in Figure 2.*

Proof. By gluing 3 cubic curves in Σ_1, we construct the trigonal curve in Σ_3 depicted in Figure 3a. By [Ore03], this implies the existence of the trigonal curve depicted in Figure 3b, with one node and one cusp. By blowing up the node of the latter curve, and blowing down the strict transform of the fiber, we obtain the curve of bidegree $(3,1)$ in Σ_2 depicted in Figure 3c, which proves the lemma. □

Lemma 3. *There exists a real algebraic curve whose Newton Polygon and chart are depicted in Figure 4.*

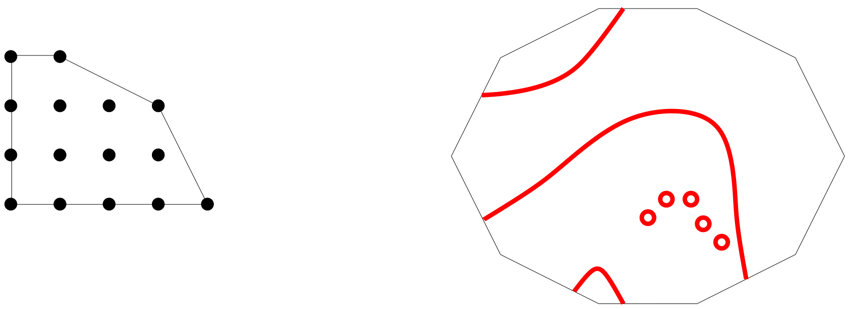
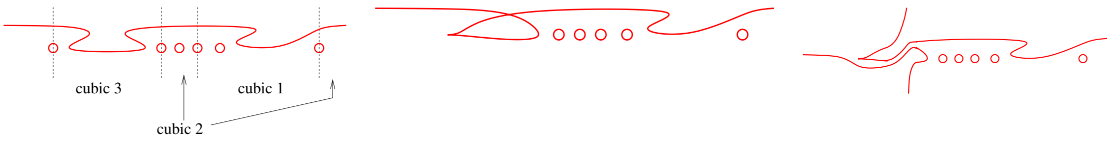


FIGURE 2



a) b) c)

FIGURE 3

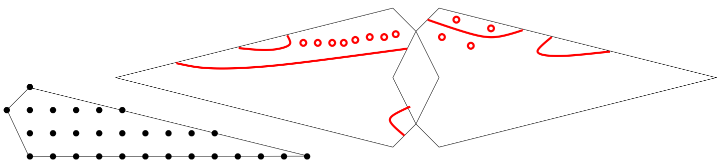
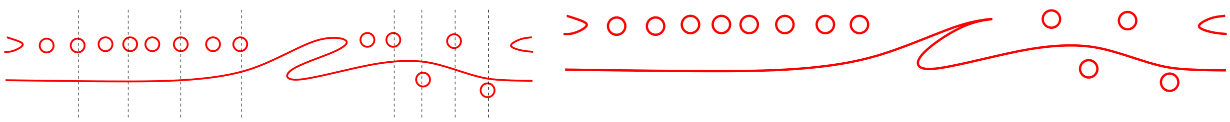


FIGURE 4

Proof. By gluing 5 cubic curves, we construct the trigonal curve in Σ_5 depicted in Figure 5a. By [Ore03], this implies the existence of the trigonal curve with one cusp depicted in Figure 5b, and prove the Lemma. □



a) b)

FIGURE 5

REFERENCES

[Che02] B. Chevallier. Four *M*-curves of degree 8. *Funct. Anal. Appl.*, 36(1):76–78, 2002.
[Ore02] S. Yu. Orevkov. A new *M*-curve of degree 8. *Funct. Anal. Appl.*, 36(3):247–249, 2002.

Date: May 14, 2011.

[Ore03] S. Yu. Orevkov. Riemann existence theorem and construction of real algebraic curves. *Annales de la Faculté des Sciences de Toulouse*, 12(4):517–531, 2003.

[Shu99] E. Shustin. Lower deformations of isolated hypersurface singularities. *Algebra i Analiz*, 11(5):221–249, 1999.

Université Pierre et Marie Curie, Paris 6, 4 place Jussieu, 75 005 Paris, France
E-mail address: brugalle@math.jussieu.fr